Calculus Warm Up #9-3

Find the point-slope equation of the line tangent to f(x) at x = 2.

$$f(x) = \frac{x + 3}{x^2 - 9}$$

Questions, p. 369

In Exercises 27–30, find the second derivative of the exponential function.

29. $g(x) = (1 + 2x)(e^{4x})$

$g'(x) = (1+2x)(e^{4x})(4) + 2e^{4x}$

$= 2e^{4x}(2+4x+1)$

$g'(x) = (2e^{4x})(4x+3)$

In Exercises 31–34, find the extrema and the points of inflection (if any exist) and sketch the graph of the function.

31. $f(x) = \dfrac{2}{1 + e^{-x}}$

f' & f''
set $= 0$
test

$f(x) = 2(1 + e^{-x})^{-1}$

$f'(x) = -2(1 + e^{-x})^{-2}(-e^{-x})$

$0 \neq \dfrac{2e^{-x}}{(1+e^{-x})^2}$ No extrema

$f''(x) = \dfrac{(1+e^{-x})^2(-2e^{-x}) - (2e^{-x})(2)(1+e^{-x})(-e^{-x})}{(1+e^{-x})^4}$

In Exercises 31–34, find the extrema and the points of inflection (if any exist) and sketch the graph of the function.

33. $f(x) = x^2 e^{-x}$

f' & f''
set $= 0$
test

$f'(x) = x^2(e^{-x})(-1) + 2x e^{-x}$

$0 = e^{-x}(2x - x^2)$ → test for extrema

$0 = x(2-x)$

$x = 0, 2$

$f''(x) = e^{-x}(2-2x) + (e^{-x})(-1)(2x - x^2)$

$e^{-x}(2 - 2x - 2x + x^2)$

$0 = e^{-x}(x^2 - 4x + 2)$ ← quadratic formula

35. Find an equation of the line normal to the graph of $y = e^{-x}$ at $(0, 1)$.

$\perp$ to tangent

$y' = -e^{-x}$

7.5 Derivatives of logarithmic functions

To find $\frac{d}{dx}\left(\ln x\right) \longrightarrow$ let $g(x) = \ln x$

we will use what we know about inverses:

$f\left(g(x)\right) = x$ if f & g are inverses

So: $\frac{d}{dx}\left[f\left(g(x)\right)\right] = \frac{d}{dx}\left[x\right]$

chain rule: $\frac{\cancel{f'\left(g(x)\right)}\left(g'(x)\right)}{\cancel{f'(g(x))}} = \frac{1}{f'(g(x))}$

Let g(x) = ln x

$g^{-1}(x) \to f(x) = e^x$

$f'(x) = e^x$

Put with our new formula:

$$g'(x) = \frac{1}{f'(g(x))}$$

$$\frac{d}{dx}(\ln x) = \frac{1}{f'(\ln x)}$$

$$= \frac{1}{e^{\ln x}}$$

Memorize

$$\frac{d}{dx}[\ln x] = \frac{1}{x}$$

THEOREM 7.9 DERIVATIVE OF THE NATURAL LOGARITHMIC FUNCTION

Let u be a differentiable function of x.

$$\frac{d}{dx}[\ln x] = \frac{1}{x}, \quad x > 0 \qquad \frac{d}{dx}[\ln u] = \frac{1}{u}\frac{du}{dx} = \frac{u'}{u}, \quad u > 0$$

Notice: Chain rule applies
and remember: you can't take the log of a negative!

1) $\frac{d}{dx}[\ln(2x)]$ $\quad u = 2x \quad \frac{du}{dx} = 2$

$$\frac{1}{\cancel{2}x} \cdot \cancel{2}$$

$$\boxed{\frac{1}{x}}$$

2) $\frac{d}{dx}[\ln(x^2 + 1)]$ $\quad u = x^2 + 1 \quad \frac{du}{dx} = 2x$

$$\frac{1}{x^2+1} \cdot 2x$$

$$\boxed{\frac{2x}{x^2+1}}$$

3) $\frac{d}{dx}[x(\ln x)]$

$$= x \cdot \frac{1}{x} + (1)\ln x$$

$$1 + \ln x$$

EXAMPLE 2 Logarithmic properties as an aid to differentiation

Differentiate

$$f(x) = \ln \sqrt{x+1}.$$

$f(x) = \ln (x+1)^{1/2}$

$f(x) = \frac{1}{2} \ln (x+1)$

$f'(x) = \frac{1}{2} \cdot \frac{1}{x+1} = \boxed{\frac{1}{2x+2}}$

EXAMPLE 3 Logarithmic properties as an aid to differentiation

Differentiate

$$f(x) = \ln [x\sqrt{1-x^2}].$$

$f(x) = \ln x + \frac{1}{2} \ln (1-x^2)$

$f'(x) = \frac{1}{x} + \frac{1}{2} \cdot \frac{1}{1-x^2} \cdot (-2x)$

$= \frac{1}{x} - \frac{x}{1-x^2}$

EXAMPLE 4 Logarithmic properties as an aid to differentiation

Differentiate

$$f(x) = \ln \frac{x(x^2+1)^2}{\sqrt{2x^3-1}}.$$

$\ln\left(\frac{x(x^2+1)^2}{\sqrt{2x^3-1}}\right)$

$f(x) = \ln x + 2\ln(x^2+1) - \frac{1}{2}\ln(2x^3-1)$

$f'(x) = \frac{1}{x} + \frac{2(2x)}{x^2+1} - \frac{1(6x^2)}{2(2x^3-1)}$

$\frac{1}{x} + \frac{4x}{x^2+1} - \frac{3x^2}{2x^3-1}$

HW: p. 392 # 1 - 37 odd

* AP Review WS # 4

due turned in tomorrow

* Group Quiz Friday: 7.2